ATAR course examination, 2018

Question/Answer booklet

MATHEMATICS APPLICATIONS

Section One: Calculator-free

Place one of your candidate identification labels in this box.

Ensure the label is straight and within the lines of this box.

Student number: In figures

In words

Number of additional answer booklets used (if applicable):

Time allowed for this section

Reading time before commencing work: five minutes
Working time: fifty minutes

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer booklet
Formula sheet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	7	7	50	55	35
Section Two: Calculator-assumed	10	10	100	96	65
				Total	100

Instructions to candidates

1. The rules for the conduct of the Western Australian external examinations are detailed in the *Year 12 Information Handbook 2018*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen. Do not use erasable or gel pens.

3. You must be careful to confine your answer to the specific question asked and to follow any instructions that are specified to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section One: Calculator-free 35% (55 Marks)

This section has **seven** questions. Answer **all** questions. Write your answers in the spaces provided.

Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

Working time: 50 minutes.

Question 1 **(4 marks)**

The weighted network below represents an orienteering map where the vertices represent the various stations and the edges represent bush tracks joining the stations. The distances on the edges are in metres. The organisers wish to install freshwater fountains at each station using the minimum length of piping necessary to connect the stations along the bush tracks.

(a) Highlight, on the diagram below, the bush tracks where the pipes should be installed.

(2 marks)
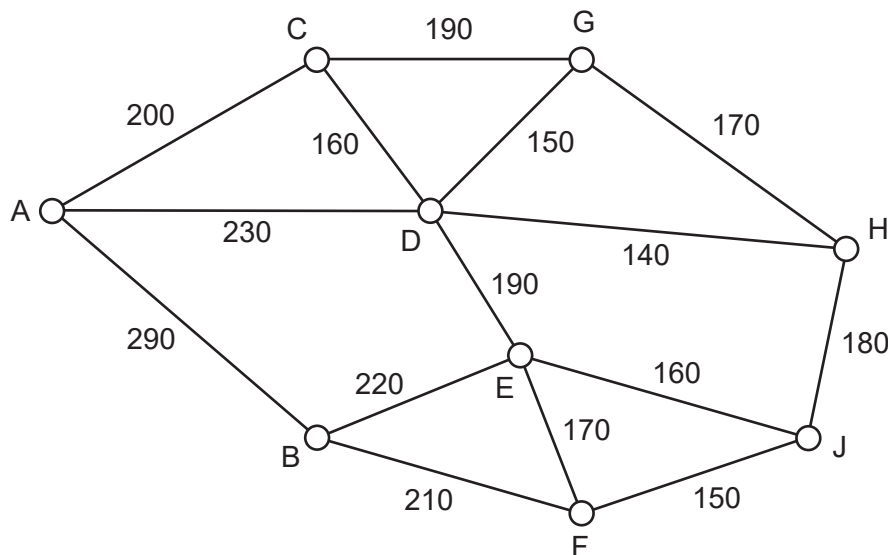
(b) Calculate the minimum length of piping required. (2 marks)

Question 2   **(8 marks)**

A bus company conducts 'jump on, jump off' sightseeing tours, during which tourists can get on and off buses at any of the designated attractions as many times as they like during the same day. The weighted digraph below shows Attractions A to F, along with the time (in minutes) that a bus takes to travel between the attractions. The bus company operates two different circuits around the city, each shown differently below.
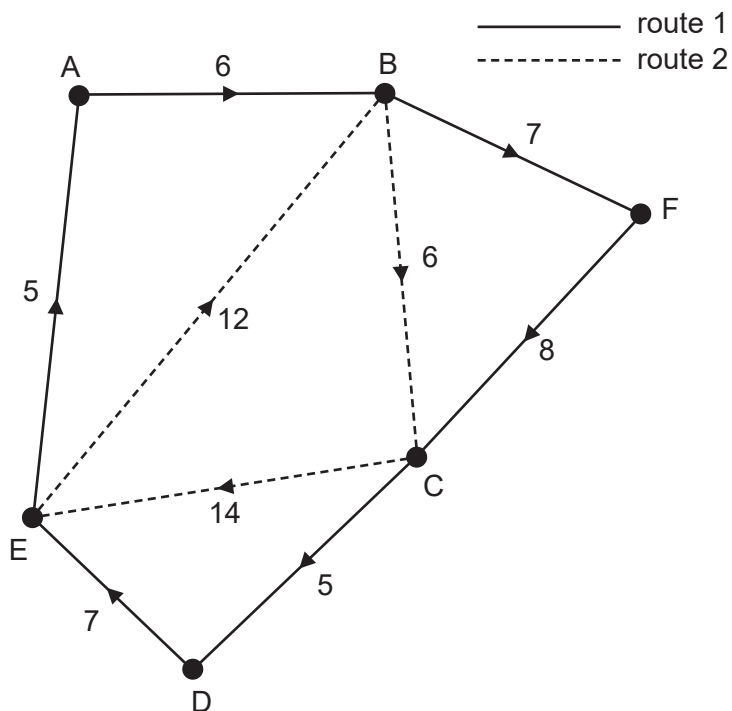

(a)   Complete the adjacency matrix for the digraph.   (2 marks)

$$
\begin{array}{c c c c c c c}
 & A & B & C & D & E & F \\
A & 0 & 1 & 0 & 0 & 0 & 0 \\
B & 0 & 0 & 1 & 0 & 0 & 1 \\
C & 0 & 0 & 0 & 1 & 1 & 0 \\
D & 0 & 0 & 0 & 0 & 1 & 0 \\
E & 1 & 1 & 0 & 0 & 0 & 0 \\
F & & & & & &
\end{array}
$$

(b)   Mai gets on a bus at E. She travels directly to B, changes bus, and continues travelling BFCDEA. Explain why her route is a trail.   (2 marks)

(c)   Vinh has just visited Attraction C and wishes to visit Attraction B next. Determine the route he should take to arrive at B in the shortest travelling time. State the time taken.

                                                                        (2 marks)

The bus company plans to include another attraction, G. Instead of adding another circuit, they will provide shuttle buses between F and G (four minutes in each direction) and D and G (five minutes in each direction), as shown below.
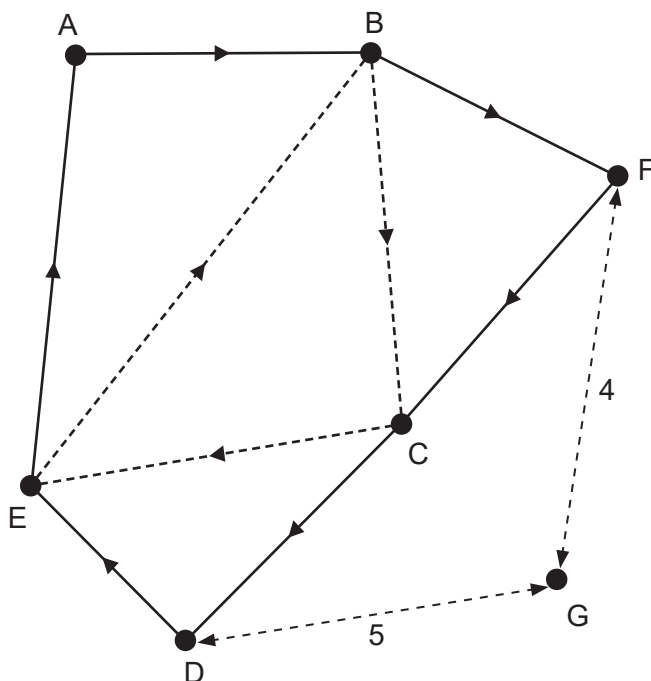
(d)   Toshi is at Attraction D. He wants to complete a Hamiltonian path. State the route he should take.

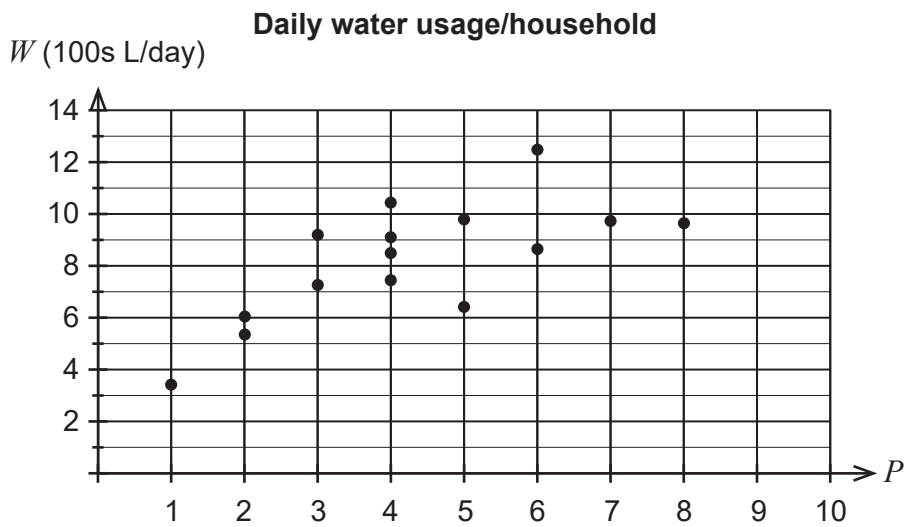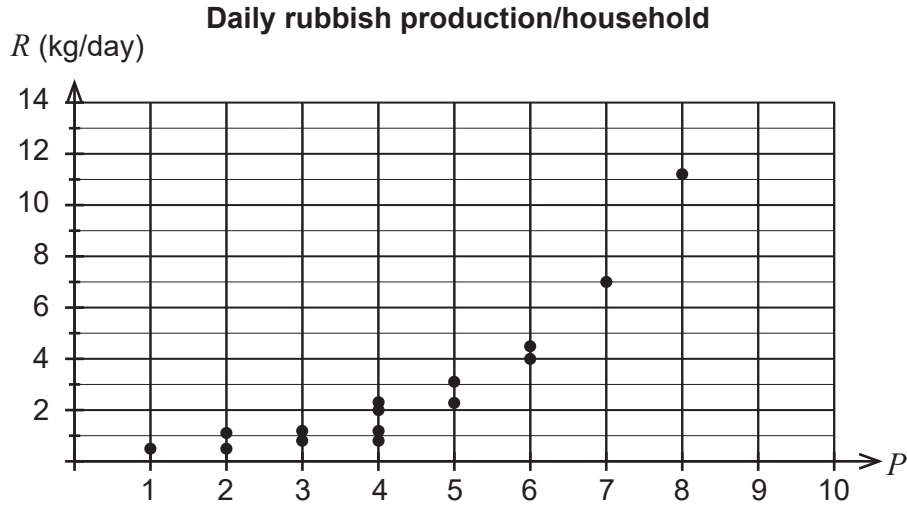   (2 marks)

Question 3   **(9 marks)**

A local council wanted to determine whether the number of people/household, P, affected the amount of household rubbish (R, kg/day) produced and/or water (W, 100s L/day) used.

The graphs below illustrate the data recorded for 15 households within the local council area.

Daily rubbish production/household

R (kg/day)

Daily water usage/household

W (100s L/day)

(a)   Describe the association between the number of people/household, P, and the daily rubbish production/household, R, in terms of strength and form.   (2 marks)

See next page

(b) It was found that approximately 49% of the variation in daily water usage, W, could be explained by the variation in the number of people/household, P. Determine the correlation coefficient (r_{PW}). (1 mark)

(c) The equation of the least-squares line for the graph showing daily water usage/household is $W = 0.83P + 4.7$.

    (i) Interpret the slope of this line. (2 marks)

    (ii) Predict the daily water usage for a household with 10 people. Comment on the likelihood of this being a valid prediction and justify your answer. (3 marks)

(d) The council argued that increasing the number of people/household causes the daily water usage to increase. Provide a non-causal explanation for the association between these two variables. (1 mark)

Question 4 (8 marks)

A company produces rolls of shade cloth. Today, there are three different machines that can be used (1, 2 and 3) and four workers who can operate these machines (Peter, Quentin, Rebecca and Sasha). Each machine will have one worker assigned to it for the whole day.

The table below shows the number of metres of shade cloth that can be produced in a day by each machine operator.

		Machines		
		1	2	3
Workers	Peter	300	250	270
	Quentin	290	410	320
	Rebecca	190	240	120
	Sasha	310	410	280

(a) Draw the weighted bipartite graph below, showing the possible allocations for each of the workers. (2 marks)

Peter   Quentin   Rebecca   Sasha
●   ●   ●   ●

●   ●   ●
Machine 1   Machine 2   Machine 3
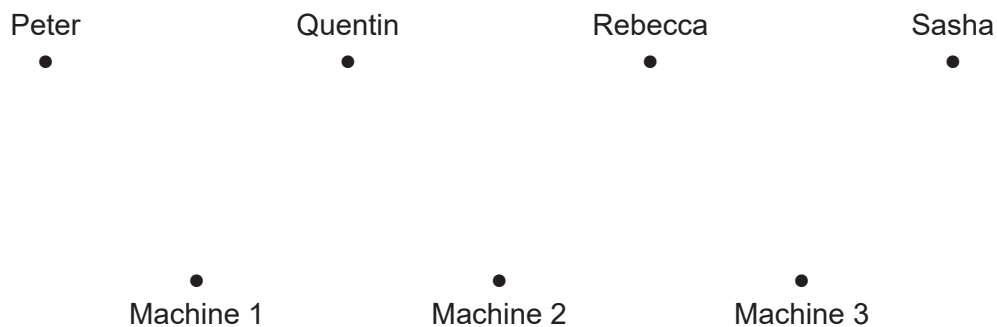
The company manager wants to allocate the workers to the machines so that the production for the day is at a maximum. She decides to use the Hungarian algorithm to determine the allocation. Her first step is to rewrite the information in matrix form, adding in a column containing all zeros.

$$\begin{bmatrix} 300 & 250 & 270 & 0 \\ 290 & 410 & 320 & 0 \\ 190 & 240 & 120 & 0 \\ 310 & 410 & 280 & 0 \end{bmatrix}$$

(b) Why has she added the column of zeros? (1 mark)

(c)   Continue the steps of the Hungarian algorithm, showing the optimum allocation of
workers to machines in the table below. State the maximum total length of shade cloth
that can be produced in the day. (5 marks)

Worker	Peter	Quentin	Rebecca	Sasha
Machine				

Maximum total length of shade cloth _____

See next page

Question 5 (9 marks)

Consider two country towns in which roads connect the local attractions. The adjacency matrix (Town 1) and graph (Town 2), shown below, represent the road connections between attractions (vertices) within each town.

Town 1

$$\begin{array}{c c} & \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \end{array} & \left[\begin{array}{cccc} 0 & 1 & 2 & 0 \\ 1 & 0 & 2 & 0 \\ 2 & 1 & 1 & 1 \\ 0 & 0 & 2 & 0 \end{array} \right] \end{array}$$

Town 2
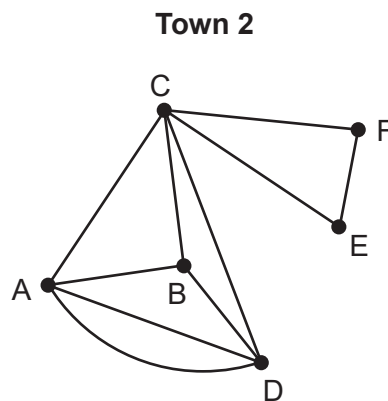
(a) Consider the adjacency matrix for Town 1.

(i) Explain why the network represented by this matrix is a directed graph. (1 mark)

(ii) Give **two** reasons why the network represented by this matrix is not a simple graph. (2 marks)

(b) Consider the graph shown for Town 2. The adjacency matrix for Town 2 has been squared, and is shown below.

$$\begin{array}{c c} & \begin{array}{cccccc} A & B & C & D & E & F \end{array} \\ \begin{array}{c} A \\ B \\ C \\ D \\ E \\ F \end{array} & \left[\begin{array}{cccccc} 6 & 3 & 3 & 2 & 1 & 1 \\ 3 & 3 & 2 & 3 & 1 & 1 \\ 3 & 2 & 5 & 3 & 1 & 1 \\ 2 & 3 & 3 & 6 & 1 & 1 \\ 1 & 1 & 1 & 1 & 2 & 1 \\ 1 & 1 & 1 & 1 & 1 & 2 \end{array} \right] \end{array}$$

See next page

    (i)   Explain the significance of the element in Row 3, Column 4.   (2 marks)

    (ii)   Draw a connected subgraph containing only vertices A, C and D.   (2 marks)

(c)   The local council of Town 2 wants to add one extra road so that an Eulerian trail is possible.

    (i)   Draw an edge on the graph below that allows this to occur.   (1 mark)
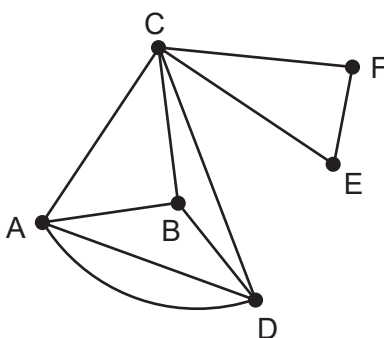
    (ii)   Explain why an Eulerian trail is now possible.   (1 mark)

Question 6 **(11 marks)**

Yana has booked three gardeners to landscape her garden. The table below shows the required activities, together with the times taken (in hours) and the immediate predecessors for each activity.

Activity	P	Q	R	S	T	U	V	W	X
Time (hours)	2.5	1.5	3	1.5	1.5	2	2.5	2	2.5
Immediate Predecessors	–	–	–	P	S, Q, U	R	T, X	P	R

(a) Complete the network diagram below, showing all tasks and durations. (3 marks)
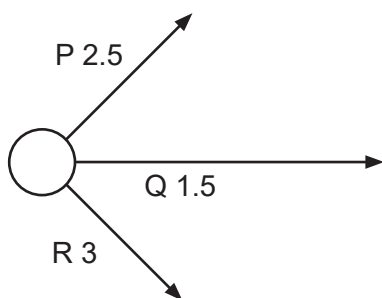
(b) Determine the critical path and the minimum completion time for the project. (2 marks)

(c) Calculate the float time for

(i) Activity W. (1 mark)

(ii) Activity U. (1 mark)

(d)   The gardeners start work at 6.30 am. They take only a half-hour break, at 12.30 pm.

(i)   Determine the latest starting time for Activity P.   (1 mark)

(ii)   Determine the earliest starting time for Activity V.   (1 mark)

(e)   One of the gardeners becomes ill and is unable to work on Yana's landscaping job.
How, if at all, will this affect the minimum completion time for this project (excluding the
gardeners' break)? Explain your answer.   (2 marks)

Question 7 **(6 marks)**

A researcher compared the performance of various golf balls. The graph below shows the height reached above the ground by a particular golf ball after each of the first three bounces. It was initially dropped from a height of 54 cm.
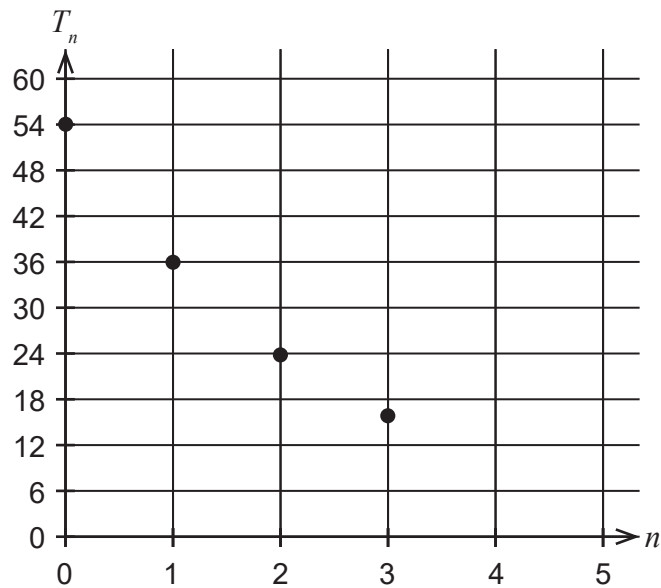
(a) Write the recursive rule for this sequence. (3 marks)

(b) Write the rule for the nth term of this sequence. (1 mark)

(c) Show that the height reached by the golf ball above the ground after the fifth bounce is $\frac{64}{9}$ cm. (2 marks)

End of section

Supplementary page

Question number: _____